

8. (a) Construct the following graphs : 8
- Eulerian but not Hamiltonian.
 - Hamiltonian but not Eulerian.
 - Neither Eulerian nor Hamiltonian.
 - Eulerian and Hamiltonian.
- (b) Define : Graph, Simple Graph, Pseudo graph and Weighted graph. 8
- (c) A tree of order n has size $(n - 1)$. Prove. 4

Roll No.

Total Pages : 4

8303

BT-3/DX

DISCRETE STRUCTURE

Paper : CSE-205(E)

Time : Three Hours]

[Maximum Marks : 100

Note : Attempt five questions in all, selecting at least one question from each section.

SECTION-I

- (a) If A , B and C be subsets of the universal set U , then prove : 14

 - $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$.
 - $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
- (b) In a city 60% of the residents can speak German and 50% can speak French. What percentage of residents can speak both the languages, if 20% residents can not speak any of these language ? 6
- (a) (i) If R be an equivalence relation defined on a non-empty set A and x, y be arbitrary elements in A , and $x \in [x]$ and $y \in [x]$, then $[x] = [y]$. 5

(ii) Prove by method of induction 5

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$$

- (b) (i) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions, then $(g \circ f)$ is one-one if both f and g are one-one and $(g \circ f)$ is onto if both f and g are onto. 5
- (ii) Give an example of a function which is (α) Injective but not Surjective (β) Bijective (γ) Surjective but not Injective, (δ) Constant. 5

SECTION-II

3. (a) Prove by constructing truth table 6
 $P \rightarrow (Q \vee R) \equiv (P \rightarrow Q) \vee (P \rightarrow R).$
- (b) Solve the recurrence relation $S_n - 7S_{n-1} + 10S_{n-2} = 0$, $S_0 = 0$ and $S_1 = 3$ by using generating function where, $n \geq 2$. 7
- (c) Find the total solution of the difference equation $S_n - S_{n-1} = 5$, given that $S_0 = 2$. 7
4. (a) Solve the difference equation $\sqrt{S_{n-1}} + \sqrt{S_{n-2}} + \sqrt{S_{n-3}} + \sqrt{\dots}$, given that $S_0 = 4$. 7
- (b) Find the total distinct numbers of six digits that can be formed with 0, 1, 3, 5, 7 and 9 and how many of them is divisible by 10? 6
- (c) Discuss the importance of recurrence relations in the binary algorithm. 7

SECTION-III

5. (a) If G is a set of Real numbers (non-zero) and let $a * b = \frac{a \cdot b}{2}$, show that $(G, *)$ is an abelian group. 7

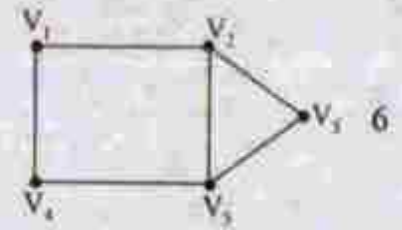
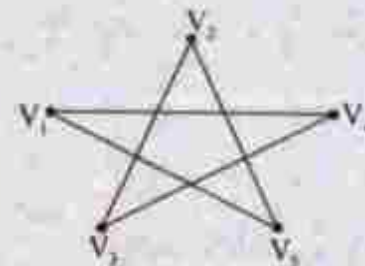
- (b) A finite integral domain is a field. Prove. 6
- (c) State and prove Lagrange's theorem. 7
6. (a) Let R is a ring with unity and $(x \cdot y)^2 = x^2 \cdot y^2 \forall x, y \in R$. Show that R is a commutative ring. 7
- (b) Show that the characteristic of an integral domain is either 0 or a prime number. 6
- (c) If H is subgroup of a group G and $h \in H$, then $Hh = H = hH$. 7

SECTION-IV

7. (a) Determine whether the graph given below by its adjacency matrix is connected or not, where the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

- (b) Find the complement of the following graphs :



- (c) If T is a binary tree of height h and order p , then $(h+1) \leq p \leq 2^{(h+1)} - 1$. 7